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New Semi-blind Channel Estimation in MIMO based on Second Order Statistics

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ABSTRACT

A new channel estimation approach for orthogonally coded multiple input multiple output (MIMO) system is proposed. The main idea of the technique proposed in this study involves the use of specific properties of the orthogonal space-time block codes (OSTBCs) in the estimation based on the second order statistic (SOS) to estimate the channel matrix after the application of singular value decomposition (SVD). This modified solution directly brings simple parameters in comparison with conventional estimation from the channel matrix. In this investigation, the authors made an attempt to reduce the effect of ambiguity which could be seen in most of the traditional blind algorithms. As the training-based least square (LS) was still used for a part of the channel matrix, the scheme in this study could be described as the semi-blind algorithm which has strongly attracted more researchers in the wireless field. A detailed analysis carried out in this current research work has validated the performance and computational advantages of the proposed method. Meanwhile, results from the simulation have confirmed the modified scheme and illustrated that the new semi-blind channel estimation is capable of improving the quality of the overall system.

Keywords: MIMO, orthogonal space-time block codes, second order statistic, semi-blind estimator

INTRODUCTION

Space time coding (STC) techniques used in the multiple input multiple output (MIMO) wireless systems are well-known method to offer significantly improved transmission rates and immunity to the fading effects as compared to early structures (Foschini & Gans, 1998; Telatar, 1999; Gesbert, *et al.,* 2003) The class of linear space-time block code (STBC) is the major category of space-time codes and it can be further divided into sub-classes like Linear Dispersion Codes, Orthogonal STBC and quasi Orthogonal STBC. Among the different space-time code schemes, the orthogonal space-time block codes (OSTBC) offer an attractive solution because they achieve full diversity at low decoding complexity. There are two advantages involved in providing transmit diversity via orthogonal designs: (1) there is no loss in bandwidth, in the sense that the orthogonal designs provide the maximum possible transmission rate at full diversity, and (2) there is an extremely simple maximum-likelihood decoding algorithm which only uses linear combining at the receiver; hence, simplicity is resulted from the orthogonality of the columns of the orthogonal design. Actually, besides decoding algorithm, the performance of MIMO systems may critically depend on the quality of the available channel state information (CSI) at the receiver. Although training-based schemes are widely used for channel estimation in MIMO as well as older techniques, a promising recent trend is to estimate the channel using efficient blind or semi-blind techniques, particularly when it is motivated by unique singular value decomposition (SVD) formulation in the matrix computation,

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whereby SVD is used to estimate channel matrices addressing simple matrix elements which are calculated using the conventional channel estimation algorithm. Many of the existing blind channel estimators, such as those based on the second-order statistics assume that the channel is static over many OSTBC blocks. After decomposition into two sub-matrices from the channel matrix, the blind method was used for the first sub-matrix, and the least square (LS) approach was utilized for the latter. Therefore, the proposed approach is the so-called semi-blind one.

On the other hand, significant progress has been observed in the blind maximum likelihood (ML) detection techniques which go along with this new channel estimation (Larsson, E. G., *et al.,* 2002). In this investigation, the blind algorithm was also modified into closed-form for simplicity in the matrix computations. Through this modified scheme, damage in term of ambiguity from the usual blind channel estimation could be reduced. The effects of the structure of the underlying OSTBC and the symbol constellation used for this modified scheme were also investigated in this study. In practice, it is difficult to find a perfect solution for this ambiguous problem, but the optimized value can be found through statistic experiments.

The paper is organized into several sections, as follows. In section II, the structure of OSTBC using in MIMO channel is introduced. In the next part (section III), the modified semi-blind channel estimation is developed. The numerical results are presented in section IV, whereas the conclusions are drawn in section V.

Notation: The bold uppercase letters denote matrices, $(.)^{\perp}$ which stands for Pseudo-inverse; $(.)^{H}$ stands for complex conjugate transpose, $\otimes$ is Kronecker product, I_N is the $N \times N$ identity matrix; $\|.\|^2$ denotes Frobenius norm; tr(.) is the sum of the elements on the main diagonal of the matrix; $vec\{.\}$ is the vectorization operator stacking all columns of a matrix on top of each other.

OVERVIEW OF ORTHOGONAL SPACE-TIME BLOCK CODE FOR MIMO

Let us first consider the block transmission scheme with the block length T, the relationship between the input and the output of the MIMO system is obtained with N transmit and M receive antennas and flat block-fading channel can be expressed as:

$$\mathrm{Y} = \mathrm{XH} + \mathrm{V} \tag{1}$$

where

$$\mathbf{Y} = \begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(T) \end{bmatrix}, \mathbf{X} = \begin{bmatrix} x(1) \\ x(2) \\ \vdots \\ x(T) \end{bmatrix}, \mathbf{V} = \begin{bmatrix} v(1) \\ v(2) \\ \vdots \\ v(T) \end{bmatrix} \tag{2}$$

are the matrices of the received signals, transmitted signals, and noise, respectively. The slow fading channel model is applied in this research work.

Besides, prior to space-time encoding, the complex information symbols are denoted as s_1, s_2, ..., s_K and that these symbols are assumed to have the properties of zero-mean mutually uncorrelated random variables which are randomly drawn from constellations U_k, $k = 1, 2, ..., K$. Let us show the vector;

$$\mathrm{s} = [s_1 s_2 \ldots s_K]^T \tag{3}$$

Also note that $s \in S$, where $S = \{\mathbf{s}^{(1)}\mathbf{s}^{(2)}...\mathbf{s}^{(L)}\}$ is the set of all possible symbol vectors and L is the cardinality of this set. The $T \times N$ matrix $\mathbf{X}(s)$ is known as OSTBC, if (Tarokh *et al.,* 1995):

- All the elements of $\mathbf{X}(s)$ are the linear functions of the K complex variables $s_1, s_2, ..., s_K$ and these complex conjugates;
- $\mathbf{X}(s)$ must satisfies the following:

$$\mathbf{X}^H(s)\mathbf{X}(\mathrm{s}) = \|\mathrm{s}\|^2\,\mathbf{I}_N \tag{4}$$

Whereas the matrix $\mathbf{X}(s)$ is defined as follows:

$$\mathbf{X}(s) = \sum_{k=1}^{K}\left(\mathbf{C}_k \operatorname{Re}\{s_k\} + \mathbf{D}_k \operatorname{Im}\{s_k\}\right) \tag{5}$$

Where, $\operatorname{Re}\{s_k\}$ and $\operatorname{Im}\{s_k\}$ denote the real and imaginary parts, respectively, and

$$\mathbf{C}_k = \mathbf{X}(e_k) \tag{6}$$

$$\mathbf{D}_k = \mathbf{X}(je_k) \tag{7}$$

with $j = \sqrt{-1}$ and e_k are respectively $K \times 1$ vector having one in its kth element and zeros elsewhere. Using equation (5), one can rewrite (1) as:

$$\underline{\mathbf{Y}} = A(\mathbf{H})\underline{s} + \underline{\mathbf{V}} \tag{8}$$

In this paper, one can arrange the matrix in real and imaginary orders. Therefore, one denotes the “underline” operator for any matrix $\mathbf{G}$ which is defined as:

$$\underline{\mathbf{G}} = \begin{bmatrix} vec\{\operatorname{Re}(\mathbf{G})\} \\ vec\{\operatorname{Im}(\mathbf{G})\} \end{bmatrix} \tag{9}$$

In most of the literature related to wireless communications, the advantage of the orthogonal feature in the calculation of matrix was used. In more specific, the algorithm in the optimized operation was applied to the $2MT \times 2K$ real matrix $A(\mathbf{H})$ with the following constraint:

$$A^T(\mathbf{H})A(\mathbf{H}) = \|\mathbf{H}\|^2\,\mathbf{I}_{2K} \tag{10}$$

Note that the columns of $A(\mathbf{H})$ have the same norms and are orthogonal to each others (Gharavi-Alkhansari & Gershman, 2003).

THE MODIFIED SEMI-BLIND CHANNEL ESTIMATION TECHNIQUE

Let consider a Rayleigh flat fading MIMO channel characterized by $\mathbf{H}$ first. Assuming that the MIMO uses N transmit and M receive antennas, the calculation is mainly based on the SVD of $\mathbf{H}$ matrix:

$$SVD(\mathbf{H}) = \mathbf{P}\Sigma\mathbf{Q}^H \tag{11}$$

Note that where $\mathbf{P}$ is a unitary matrix, the matrix Σ is a diagonal matrix with non-negative real numbers on the diagonal, and $\mathbf{Q}$ is also a unitary matrix. Both $\mathbf{P}$ and $\mathbf{Q}$ must satisfy the following properties:

$$\mathbf{P}\mathbf{P}^H = \mathbf{P}^H\mathbf{P} = \mathbf{I} \quad \text{and} \quad \mathbf{Q}^H = \mathbf{Q}^H\mathbf{Q} = \mathbf{I} \tag{12}$$

Based on the previous literature, many methods are applied for estimating the matrix **Q**, i.e. the estimation of **Q** based on the training pilots. From these decompositions, the whitening matrix **W** is given by **W = P**Σ, which can be estimated blindly, and particularly based on the received data only. Interestingly, if **P** matrix is orthogonal, the **W** matrix is also orthogonal. The estimation of **W** matrix can be done by using high order statistic, but this particular technique need larger number of transmitted data to obtain the exact estimated channel. In this investigation, the channel estimation of **W** matrix from the second order statistic was used, as it has been proven to be more effective than the other candidates, with the assumption of **Q** matrix known perfectively. In addition, some theoretical results have shown that the estimation of the channel matrix based on estimation **W**, **Q** can perform more efficiently than estimating **H** directly from the pilot data. This is because the orthonormal matrix **W** was applied and this led to use lesser number of parameters than the complete channel matrix **H**, and hence, it could be estimated with greater accuracy from the limited pilot data.

Let us introduce the $2MN \times 1$ channel vector **W,** as:

$$\mathbf{W} = \underline{\mathbf{W}} \tag{13}$$

When A(**W**) is linear in **W**, there exists a unique $4\,KMT \times 2MN$ matrix Φ, such that:

$$vec\{A(\mathbf{W})\} = \mathbf{\Phi W} \tag{14}$$

The key feature in this paper is based on the orthogonal matrix calculation. From equations (10) and (13), the following was obtained:

$$A^T(\mathbf{W})A(\mathbf{W}) = \|\mathbf{W}\|^2\, \mathbf{I}_{2K} \tag{15}$$

Applying the trace operator to both sides of (15), the following was retrieved:

$$\mathbf{W}^T\mathbf{\Phi}^T\mathbf{\Phi W} = 2K\,\|\mathbf{W}\|^2 \tag{16}$$

As equation (16) is satisfied for any **W**, it leads to the following expression:

$$\mathbf{\Phi}^T\mathbf{\Phi} = 2K\mathbf{I}_{2K} \tag{17}$$

Through expression (17), it can be seen that the columns of the **Φ** are orthogonal to each other.

Before introducing the blind channel estimation, the following Lemma (Manton, 2002) is needed:

Lemma 1: Let **B** be an $m \times q$ real matrix, where $q \leq m$; for any $m \times m$ real symmetric matrix **M**, the solution to the following optimization problem:

$$\begin{aligned} &\max_B tr\left\{\mathbf{B}^T\mathbf{MB}\right\} \\ &s.t.\mathbf{B}^T\mathbf{B} = \mathbf{I}_b \end{aligned} \tag{18}$$

is given by any matrix $\mathbf{B}_*$ whose column space is the same as the subspace spanned by the b principal eigenvector of **M** and, for any such $\mathbf{B}_*$

$$tr\left\{\mathbf{B}_*^T\mathbf{MB}_*\right\} = \sum_{i=1}^{b} v_i \tag{19}$$

where v_i, $i = 1, 2, ..., b$ are the b largest eigenvalues of **M**
Proof: See (Manton, 2002)

Next, the following set of the parameters is available:

$$\begin{aligned} &b = 2K \\ &\mathbf{B} = A(\tilde{\mathbf{W}}) / \| \tilde{\mathbf{W}} \| \\ &\mathbf{M} = E\{\underline{\mathbf{Y}}\,\underline{\mathbf{Y}}^T\} = A(W)E\{\underline{s}\,\underline{s}^T\}A^T(\mathbf{W}) + (\sigma^2/2)\mathbf{I}_{2MT} \end{aligned} \tag{20}$$

Applying the Lemma above and based on equation (18), the optimized constraint was obtained to estimate matrix **W** as follows:

$$\max_{\tilde{W}} \frac{tr\{A^T(\tilde{\mathbf{W}})\}MA(\tilde{\mathbf{W}})}{\| \tilde{\mathbf{W}} \|^2} \tag{21}$$

Next, decompose the optimized equation into some elements which are calculated easily, as follows:

$$\begin{aligned} &tr\{A^T(\tilde{\mathbf{W}})\mathbf{M}A(\tilde{\mathbf{W}})\} = vec\{(\tilde{\mathbf{W}})\}^T(\mathbf{I}_{2K} \otimes \mathbf{M})vec\{A(\tilde{W})\} \\ &= (\tilde{\mathbf{W}})^T \mathbf{\Phi}^T (\mathbf{I}_{2K} \otimes \mathbf{M})\mathbf{\Phi}\tilde{\mathbf{W}} \end{aligned} \tag{22}$$

Therefore, the optimization issue (21) will be changed, as follows:

$$\max_{\tilde{W}} \frac{(\tilde{\mathbf{W}})^T \mathbf{\Phi}^T (\mathbf{I}_{2K} \otimes \mathbf{M})\Phi\mathbf{M}\tilde{\mathbf{W}}}{\| \mathbf{W} \|^2} \tag{23}$$

For simplicity purposes, $\mathbf{\Phi}^T(\mathbf{I}_{2K} \otimes \mathbf{M})\mathbf{\Phi} = \mathbf{U}$, and a new optimization expression is obtained (note that the estimation of **W** matrix is required).

$$\max_{\tilde{W}} \frac{(\tilde{\mathbf{W}})^T \mathbf{U}\tilde{\mathbf{W}}}{\| \tilde{\mathbf{W}} \|^2} \tag{24}$$

In particular, note that all the solutions to this optimization depend on the sub-spaces spanned by the n linearly independent principle eigenvectors of the **U** matrix, with n as the multiplicity order of the largest eigenvalue of this matrix. The value of n depends on the matrix $\mathbf{\Phi}$, which in turn, is dependent on the structure of the underlying OSTBC (Tarokh *et al.,* 1999). It is difficult to find the relationship between the value of n and the structure of $\mathbf{\Phi}$. Through the experiments, one can also observe that in most cases tested, $n = 1$ is always true. Ignoring the scaling ambiguity, the normalized solution to (23) can be illustrated as:

$$\hat{\tilde{\mathbf{W}}} = \Omega(\mathbf{U}) \tag{25}$$

where $\Omega(\mathbf{U})$ stands for the normalized principle eigenvector of a matrix $\|\Omega(\mathbf{U})\|$, and $\hat{\tilde{\mathbf{W}}}$ is **W** estimated matrix. In addition, the least square estimation of the **Q** matrix, using only the pilot data, is given by:

$$\hat{\mathbf{Q}}_{LS} = \mathbf{Y}\mathbf{X}^{\perp} \tag{26}$$

where $\hat{\mathbf{Q}}_{LS}$ is the estimated matrix of **Q**, following the least square algorithm.

Finally, combining the two estimated channel matrices (both **W** and **Q** matrices mentioned above), a complete estimated channel estimation known as the **H** matrix is obtained.

SIMULATION RESULTS

In all the simulations, the performance of the proposed approach for semi-blind channel estimation is presented. Some basic parameters which have been used in this investigation are: modulation format of QPSK, 1000 Monte Carlo trials, a Rayleigh distributed channel, indicating that each element of **H** channel follows a complex Gaussian circular law with zero-mean and unit variance, and the transmitted symbols are i.d.d. on every antenna. Moreover, a spatially white Gaussian noise is used in these simulations. The rate of the OSTBC is defined as the ratio between the numbers of symbols the encoder takes as its input and the number of space-time coded symbols transmitted from each antenna.

The authors analysed the performance of the algorithm presented in this paper with the results from the research work of Ammar *et al.* in 2007. From *Fig. 1*, the lines in term of MSE criterion collides with each other, proving that both solutions with the same quality. Besides, the extremely high complexity in blind channel estimation is determined as the reason for slow convergence at high SNR parameter.

Based on the research work of Tarokh *et al.* in 1999*,* some orthogonal schemes were applied for the MIMO system with *t* transmit and 4 receive antennas, such as OSTBC3, OSTBC4, OSTBC5, and OSTBC6, with the number of transmit antennas as 3, 4, 5, 6 respectively. In this experiment, the rate of 1/2 is set, and the number of symbols to be transmitted is 512. In the second experiment (*Fig. 2*), it can be seen that the performance is better as fewer number of transmit antennas is used, with the assumption of a fixed number of receive antennas.

In the third experiment (*Fig. 3*), the simulation for MIMO OSTBC3 is displayed with a code rate of ¾ for the different numbers of transmitted signal. Intuitively, the solution proposed stably remains through a large number of symbols. This illustration proves that applying the SOS-based

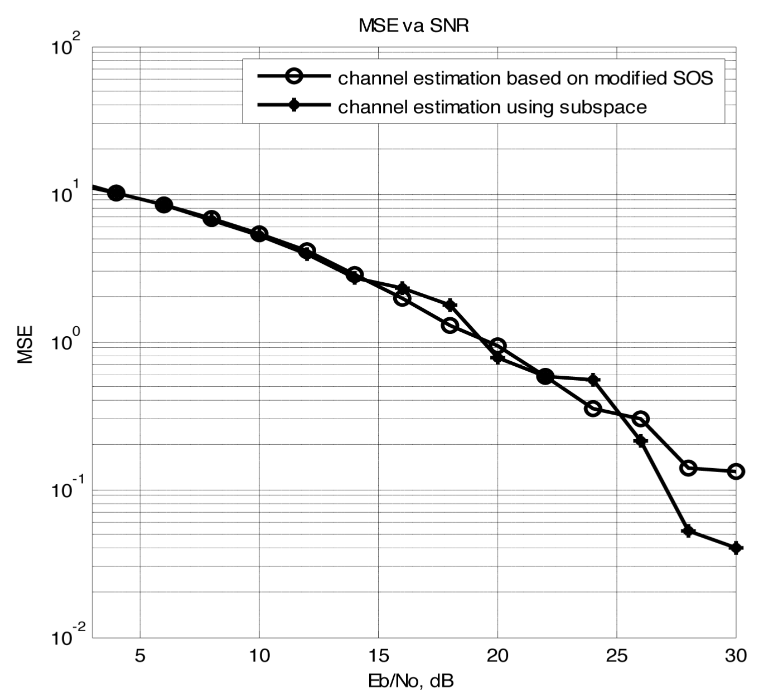

Fig. 1: The comparison between the proposed method and one using the subspace

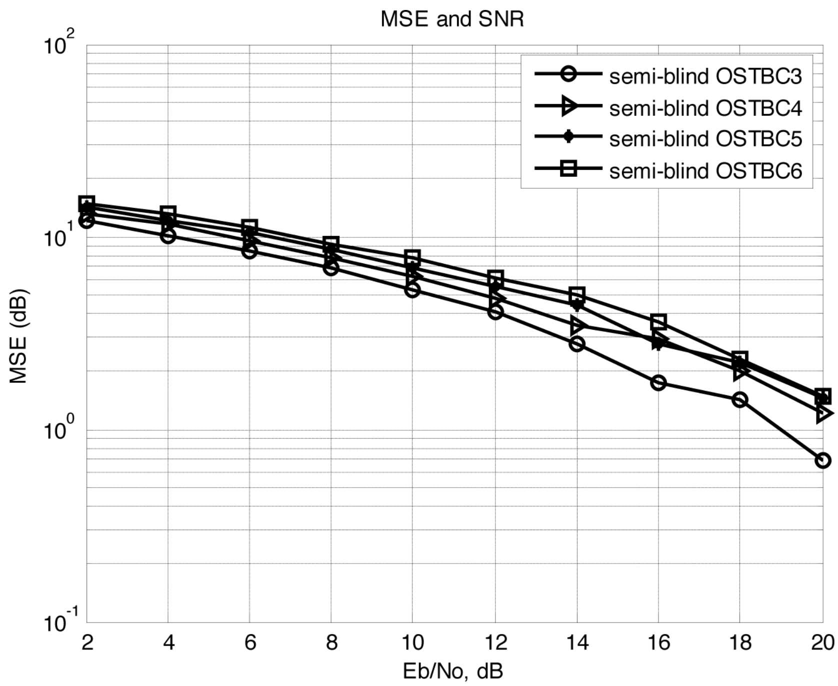

Fig. 2: The MSE performance versus SNR

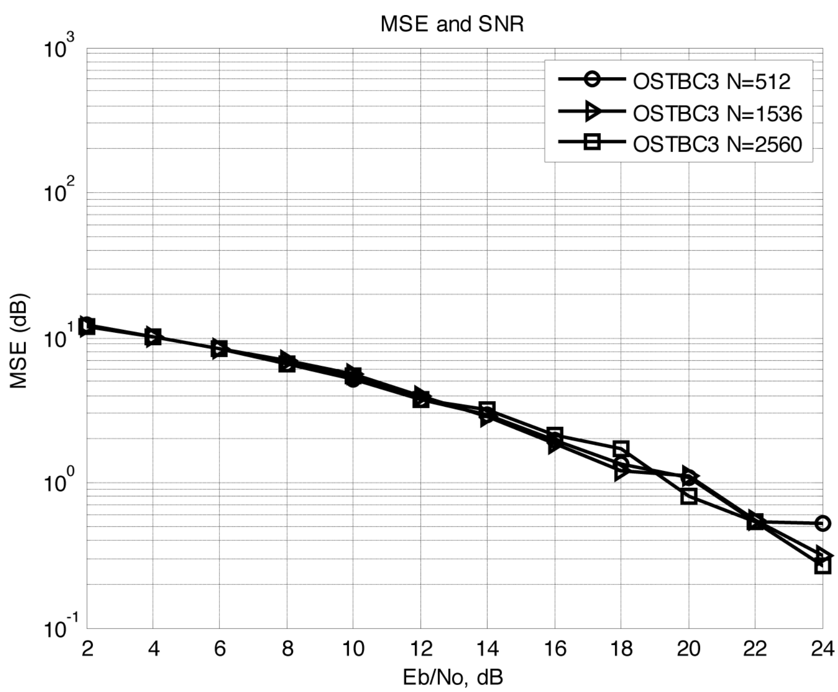

Fig. 3. The MSE performance versus SNR with different numbers of transmitted symbols (N)

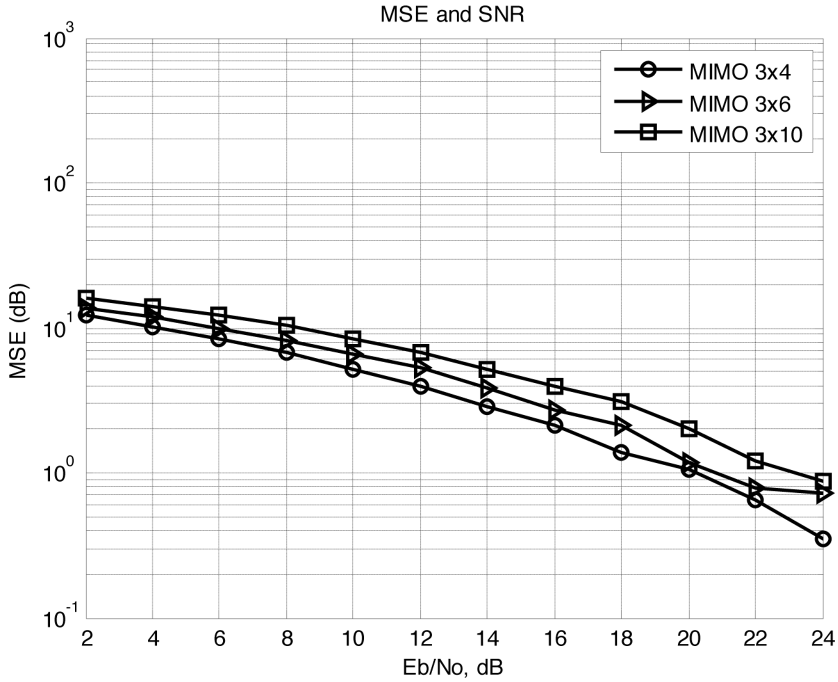

Fig. 4: The MSE performance versus SNR with different MIMO OSTBC3 systems

channel estimation for the detached matrix channel (based on the SVD calculation) leads to the optimization technique of the MIMO deployment in the reality.

In the rest of the experiments, *Fig. 4* shows that the performance of MIMO OSTBC3, with 3 transmits antennas and 4, 6, 10 receive antennas, respectively. In particular, MIMO ¾ is considered as the best performance among the three experimental approaches. It is true that balancing the number of transmits and receives antennas which help to create the improving scene. Therefore, this result also proves that the MIMO with a decreasing cost can bring a substantial capability in the wireless applications because of fewer numbers of designed antennas.

CONCLUSION

In this study, the new semi-blind channel estimation has been presented by applying both the second order statistics and the training-based least square techniques which address the use of the properties of the orthogonal matrix columns in the transmitted signals. Through the paper, it is true that the structure of the OSTBC effect on the estimated channel is carefully calculated especially in the different MIMO systems for higher data rate. The researchers have attempted to solve the ambiguity problem in previous literature, particularly in term of blind estimation. The MSE analyses in the simulations are clear demonstrations which have provided the improvement work for channel estimators.

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